Measuring contagion risk in international banking

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Objective

- Improve the measurement of **credit risk** of a country
- taking into account **capital flows** from a country banking system to:
  - other banking systems
  - other public sectors
  - other non-bank private sectors

Contribution

1. The reference literature:
   - **Network models**: Battiston et al. (2012), Billio et al. (2012), Diebold and Ylmaz (2014)
   - **Network models in international banking**: Mc Guire and Tarashev (2006), Von Peter (2007) Minoiu and Reyes (2013), Giudici and Spelta (2016, 2017)
2. Our contribution:
   - Introduces **multilayer network models** in international banking
   - Improves **credit risk prediction**, correcting CDS spreds with contagion
   - Provides an early warning **measure of contagion**, in terms of expected losses, on the borrowing and on the lending side

Data - I

- We obtain data on banks' foreign exposures from BIS Consolidated Banking Statistics (CBS) on an ultimately risk (UR) basis.
- Quarterly panel (from Q1 2005 to Q4 2015)
- At each point in time, 3 cross-sectional dimensions: 23 x 23 x 3 matrix at each t:
  - 23 national banking systems
  - 23 borrowing countries
  - 3 counterparty sectors

Data - II

Three sets of CDS spreads series – one for each of the three counterparty sectors:

- Public sector: five-year sovereign on-the-run CDS spread (Markit)
- Bank sector: equally-weighted average of senior five-year CDS spreads (Markit)
- Non-bank private sector: option-adjusted spreads between yield on country corporate index and US Treasuries (Barclays Global Aggregate Corporate Index and JPM's CEMBI)

Multilayer networks

- **Multilayer networks** describe multidimensional interlinkages between economic agents (Aldasoro and Alves 2016, Montagna and Kok 2016, Poledna et al. 2015).
- In our extension, a multilayer network represents the **amount** of each national banking system's foreign claims along the three layer types (Bank, Official and Non bank private sectors).
- The network can be mapped to a 3-way tensor $\mathcal{X} \in \mathbb{R}^{I \times I \times K}$ where I represents **countries** and K the **type** of exposures (borrowing sectors).
- We let a generic element of the tensor x_{ijk} be the **share** of funds borrowed by country i (authority) from national banking system j (hub) in type k, with respect to the total foreign claims.

Probabilistic tensor decomposition - I

The elements of a tensor can be normalised into conditional frquencies:

$$h_{i|jk} = \frac{x_{ijk}}{\sum_{j=1}^{J} \sum_{k=1}^{K} x_{ijk}} \quad i = 1, \ldots, I$$

$$a_{j|ik} = \frac{x_{ijk}}{\sum_{i=1}^{I} \sum_{k=1}^{K} x_{ijk}} \quad j = 1, \ldots, J$$

$$r_{k|ij} = \frac{x_{ijk}}{\sum_{i=1}^{I} \sum_{j=1}^{J} x_{ijk}} \quad k = 1, \ldots, K$$

which can be used to estimate the transition probabilities of a Markov Chain:

$$\Pr\left[X_\tau = i | Y_\tau = j, Z_\tau = k\right]$$

$$\Pr\left[Y_\tau = j | X_\tau = i, Z_\tau = k\right]$$

$$\Pr\left[Z_\tau = k | Y_\tau = j, X_\tau = i\right].$$

Probabilistic tensor decomposition - II

Define:

$$\Pr\left[X_\tau = i\right] = \sum_{j=1}^{J} \sum_{k=1}^{K} h_{i|jk} \Pr\left[Y_\tau = j, Z_\tau = k\right]$$

$$\Pr\left[Y_\tau = j\right] = \sum_{i=1}^{I} \sum_{j=1}^{K} a_{j|ik} \Pr\left[X_\tau = i, Z_\tau = k\right]$$

$$\Pr\left[Z_\tau = k\right] = \sum_{i=1}^{I} \sum_{j=1}^{I} r_{k|ij} \Pr\left[Y_\tau = j, X_\tau = i\right].$$

Their limiting distributions:

$$u_i = \lim_{\tau \to \infty} \Pr\left[X_\tau = i\right]$$

$$v_j = \lim_{\tau \to \infty} \Pr\left[Y_\tau = j\right]$$

$$w_k = \lim_{\tau \to \infty} \Pr\left[Z_\tau = k\right],$$

will be used as hub, authority and type **probability** scores.

Probabilistic tensor decomposition - III

In practice, scores can be derived solving the following system of equations:

$$u_i = \sum_{j=1}^{I} \sum_{k=1}^{K} h_{i|jk} v_j w_k \quad i = 1, ..., I$$

$$v_j = \sum_{i=1}^{I} \sum_{k=1}^{K} a_{j|jk} u_i w_k \quad j = 1, ..., I$$

$$w_k = \sum_{i=1}^{I} \sum_{j=1}^{I} r_{k|ij} u_i v_j \quad k = 1, ..., K$$

iteratively until $\left\|\mathbf{u}^{(\tau)} - \mathbf{u}^{(\tau-1)}\right\| + \left\|\mathbf{v}^{(\tau)} - \mathbf{v}^{(\tau-1)}\right\| + \left\|\mathbf{w}^{(\tau)} - \mathbf{w}^{(\tau-1)}\right\| < \epsilon.$

Ng, Li and Ye (2011) show that such solution exists and it is unique.
$\mathcal{X}$ can then be approximated by the (probabilistic) adjacency matrix:

$$\mathbf{M} = \mathbf{uv}.$$

Probabilistic tensor decomposition - IV

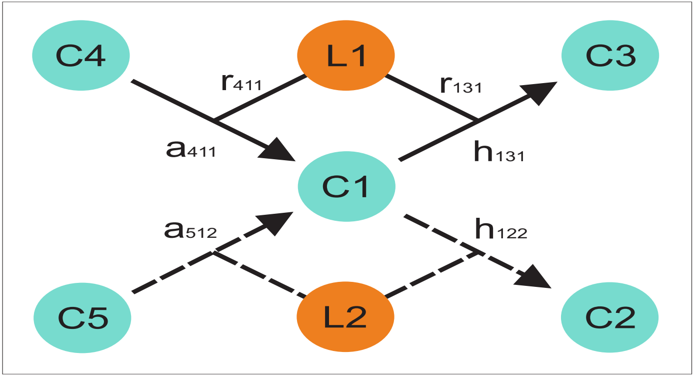


Figure: The authority score of C_1 is $u_{C_1} = a_{411} v_{C_4} w_{L_1} + a_{512} v_{C_5} w_{L_2}$. The hub score of C_1 is $v_{C_1} = h_{131} v_{C_3} w_{L_1} + h_{122} v_{C_2} w_{L_2}$. The type score of L_1 is $w_{L_1} = r_{411} v_{C_4} u_{C_1} + r_{131} v_{C_1} u_{C_3}$

Network contagion - I

In Network models, the alpha-centrality vector $\mathbf{S}$ is the steady state solution of the equation:

$$\mathbf{S}_\mu = \mathbf{C} + \beta\, \mathbf{A}\, \mathbf{S}_{\mu-1}$$

or, equivalently:

$$\mathbf{S}_\mu = \left(\mathbf{I} + \beta \mathbf{A} + \beta^2 \mathbf{A}^2 + \ldots + \beta^\mu \mathbf{A}^\mu \right) \mathbf{C},$$

where $\mathbf{C}$ is a vector of positive quantities, $\mathbf{A}$ an adjacency matrix, and

$$\beta = \frac{\alpha}{\lambda_1 (\mathbf{A})},$$

with $\lambda_1(A)$ the largest eigenvalue of $\mathbf{A}$ and $0 \leq \alpha \leq 1$.

Network contagion - II

- In our context, c^{ik} is the CDS spread of country i in type k, and $\mathbf{C}^k$ is the vector of all spreads of type k. We then take $\mathbf{C} = \frac{1}{k} * \sum_{k=1}^{K} \mathbf{C}^k$.
- The matrix $\mathbf{A}$ can express "exposure" to contagion, letting $\mathbf{A} = \mathbf{M}$
- Last, concerning the rate of contagion α, we assume that:

$$\alpha^o = w_o, \alpha^b = w_b, \alpha^p = w_p$$

Network contagion - III

As in Tintchev (2016) we assume that contagion can come both from the lending side (credit risk) and on the borrowing side (funding risk). As a country can borrow only from the banking sector of another country, on the borrowing side we let $\mathbf{C} = \mathbf{C}^b$.

We then define a **multivariate spread**, respectively on the borrowing (SB) and on the lending side (SL), as:

$$\mathbf{SB} = \mathbf{C} + \mathbf{C}^b \sum_{\mu=1}^{\Delta} \beta^{b,\mu} \mathbf{M}^{\mu} + \mathbf{C}^b \beta^o \mathbf{M} + \mathbf{C}^b \beta^p \mathbf{M}$$

$$\mathbf{SL} = \mathbf{C} + \mathbf{C}^b \sum_{\mu=1}^{\Delta} \beta^{b,\mu} \left(\mathbf{M}^T\right)^{\mu} + \mathbf{C}^o \beta^o \mathbf{M}^T + \mathbf{C}^p \beta^p \mathbf{M}^T,$$

where b, o, p indicate the bank, official and non bank sectors.

Expected losses

- Let E to be the **total foreign claims** of the system at a given time point.
- The **expected loss** for a country i, on the **borrowing** side, can be obtained as:

$$EL_i^B = E * \sum_j \mathbf{M}_{ij} * SB_j.$$

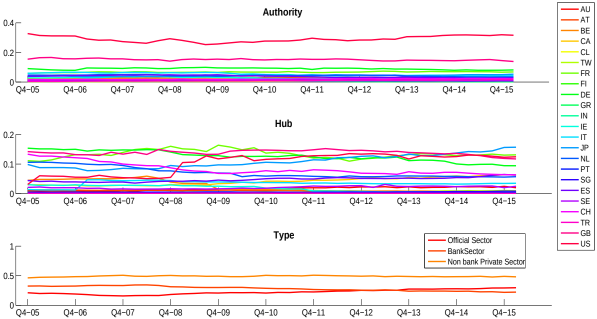
- The **expected loss** for a country i, on the **lending** side, can be obtained as:

$$EL_i^L = E * \sum_j \mathbf{M}_{ij}^T * SL_j$$

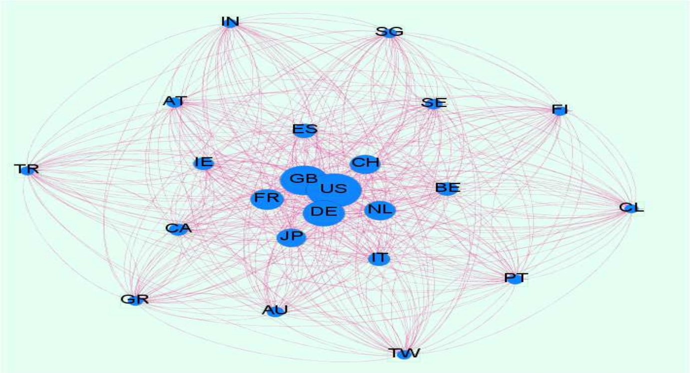
Predictive performance - I

- We compare our multivariate spreads with the market CDS spreads, in terms of one-step ahead predictive accuracy.
- Specifically, we compare: i) a model based only on the average of the last n_1 observed past spread values; ii) a model based on the same observed past spreads, modified by network contagion.
- For both models, we compare the predictive Root Mean Squared Error, for different n_1 lengths: $N = 1, \ldots, 8$ quarters.
- We also compare their binarised versions, in which we set a performance indicator equal to 1 when the RMSE of i) is higher than that of ii) and zero otherwise.

Hub, Authority and Type Scores



The Probabilistic Network

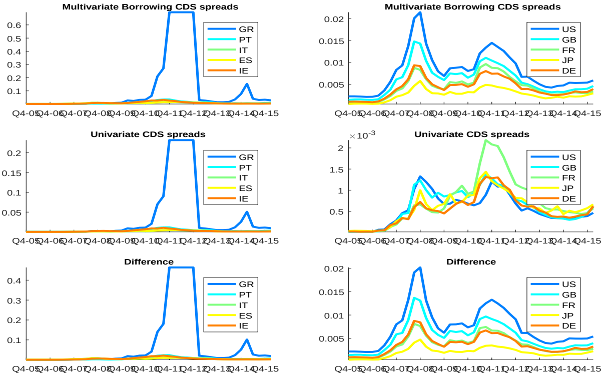


Summary Statistics

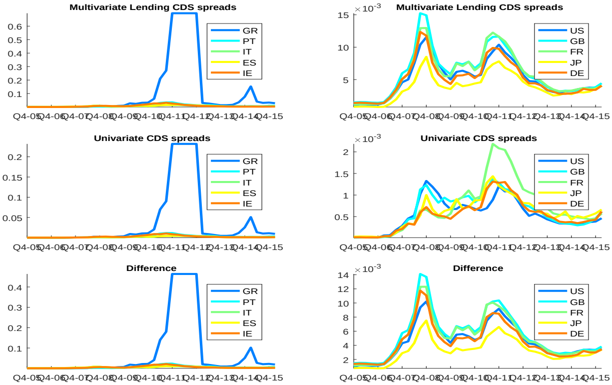
	Market Cds	Multivariate Borrowing	Multivariate Lending
GR	0.0373	0.1120	0.1119
PT	0.0030	0.0092	0.0091
IT	0.0016	0.0064	0.0062
ES	0.0017	0.0064	0.0066
IE	0.0025	0.0081	0.0080
US	0.0006	0.0080	0.0051
GB	0.0006	0.0060	0.0060
FR	0.0007	0.0044	0.0058
JP	0.0006	0.0026	0.0038
DE	0.0005	0.0042	0.0050

TABLE: Mean CDS spreads: market (univariate) and enhanced (multivariate)

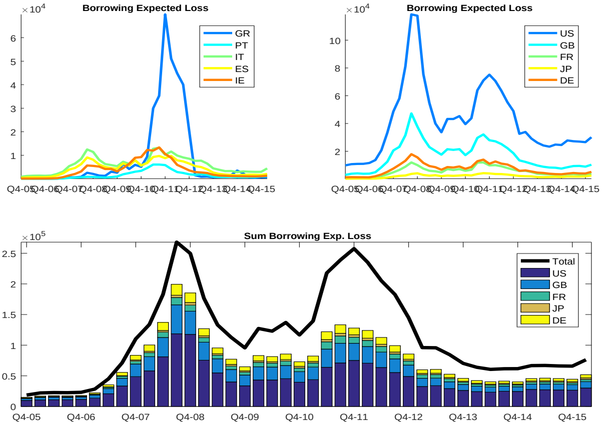
Multivariate Borrowing Spread



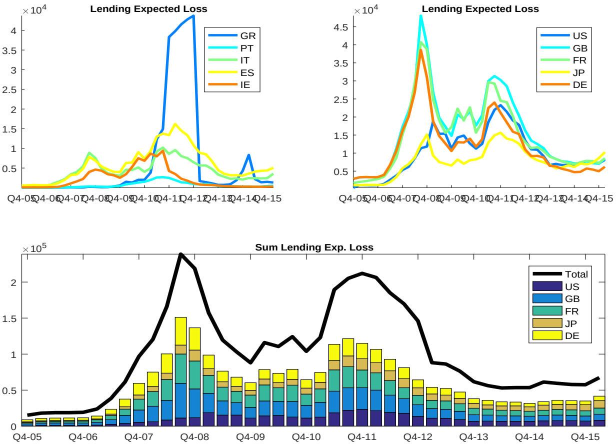
Multivariate Lending Spread



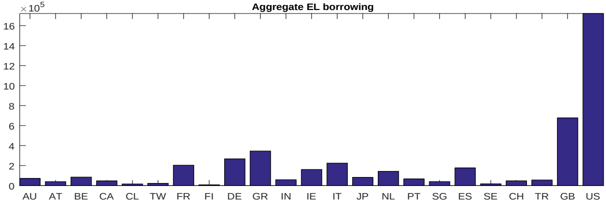
Borrowing EL

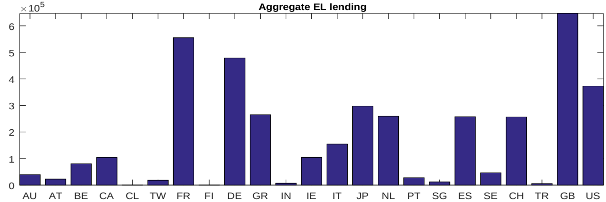


Lending Expected Loss

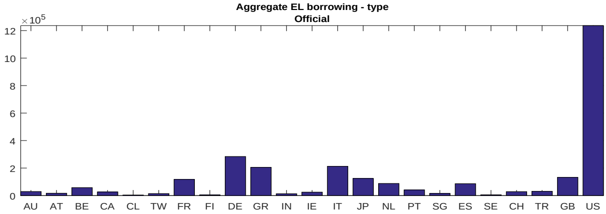


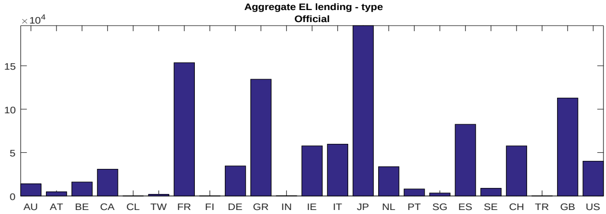
Borrowing vs Lending: all sectors' cumulated expected losses



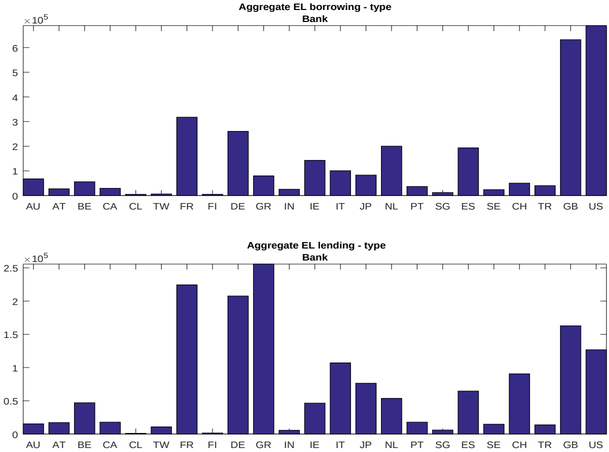


Borrowing vs Lending: official sector's cumulated expected losses

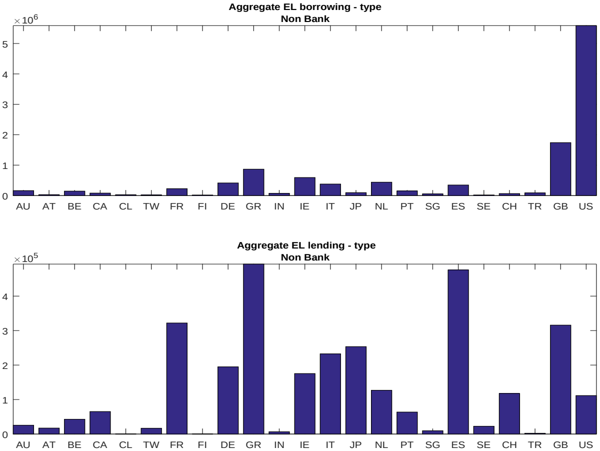




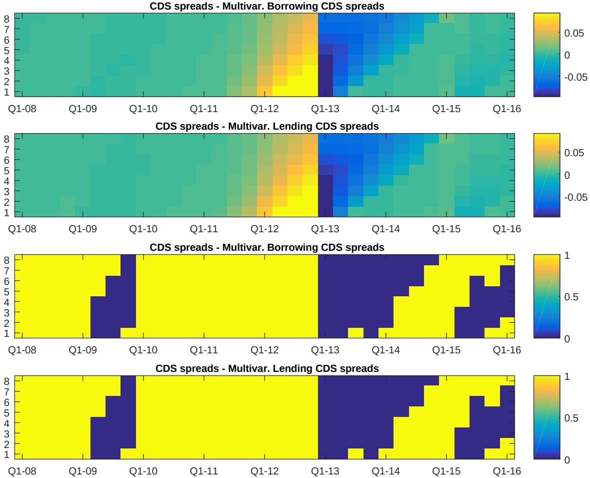
Borrowing vs Lending: bank sector's cumulated expected losses



Borrowing vs Lending: non bank private sector's cumulated expected losses



Predictive performance



Conclusions

- Our aim is to measure **contagion risk** in international banking.
- To achieve this aim we have developed a **multilayer network** model, based on a probabilistic tensor decomposition and on a multivariate contagion mechanism.
- The model provides **"augmented" CDS spreads** monitors, that improve stress predictions, especially during crisis times.
- It can also monitor the **expected losses** of each banking system, on the borrowing or on the lending side, at any given time point.

Further research

- Network models to measure contagion risk in **fintech P2P lending** (with Branka Hadji-Misheva)
- Marketplace P2P lending disintermediates risks: may increase volumes underestimating risks. Empirical testing in progress.
- P2P lending involves higher interconnecdness: this may increase contagion but may improve risk measurement (through network models). Data acquisition in progress.