Market Risk and the FRTB (R)-*Evolution*

Review and Open Issues

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A Market Risk

The Basel Comittee supervisory approach requires that:

- The banks **measure** their own risks
- The banks must satisfy the rule

Regulatory Capital > Risks

(more often scaled to Capital / RWA > 8%, where Risks = RWA x 8%)

How to measure the risks? 2 possibles techniques:

- *Standard models*, i.e. grids of coefficients to apply to the exposures
- *Internal Models*, that rely on statistical figures (metrics) in order to capture the risk magnitude with a conservative approach. They are approved by the Central Bank after a very complex *validation* process, concerning statistical properties, the calculation trackability, the ICT systems and so on

Let us give some practical examples

Standard Models

- Instrument (or risk factor) opposite positions *off-set*
- **Equity Positions**. For a cash position: 8% as a provision for *generic* risk, 2-4% as a provision for the *specific* risk
- **Interest rate positions**. Maturity (Or Duration) buckets, hence application of the coefficients of the below list
- **Derivatives** positions.
  - *Delta-Plus* approach
  - The *Delta-Gamma-Vega* greeks are needed
  - To the **Delta-Gamma** exposures the usual coefficients are applied (e.g. 8%)
  - **Vega**. A *relative* shock of **25**% to the current volatility is applied

Tabella 1: Metodo basato sulla scadenza: fasce temporali e fattori di ponderazione

Zone	Fasce temporali di scadenza		Fattori di ponderazione
	cedola pari o superiore al 3%	*cedola inferiore al 3%*	
Zona 1	fino a 1 mese	fino a 1 mese	0 %
	da oltre 1 mese fino a 3 mesi	da oltre 1 mese fino a 3 mesi	0,20 %
	da oltre 3 mesi fino a 6 mesi	da oltre 3 mesi fino a 6 mesi	0,40 %
	da oltre 6 mesi fino a 1 anno	da oltre 6 mesi fino a 1 anno	0,70 %
Zona 2	da oltre 1 anno fino a 2 anni	da oltre 1 anno fino a 1,9 anni	1,25 %
	da oltre 2 anni fino a 3 anni	da oltre 1,9 anni fino a 2,8 anni	1,75 %
	da oltre 3 anni fino a 4 anni	da oltre 2,8 anni fino a 3,6 anni	2,25 %
Zona 3	da oltre 4 anni fino a 5 anni	da oltre 3,6 anni fino a 4,3 anni	2,75 %
	da oltre 5 anni fino a 7 anni	da oltre 4,3 anni fino a 5,7 anni	3,25 %
	da oltre 7 anni fino a 10 anni	da oltre 5,7 anni fino a 7,3 anni	3,75 %
	da oltre 10 anni fino a 15 anni	da oltre 7,3 anni fino a 9,3 anni	4,50 %
	da oltre 15 anni fino a 20 anni	da oltre 9,3 anni fino a 10,6 anni	5,25 %
	oltre 20 anni	da oltre 10,6 anni fino a 12 anni	6,00 %
		da oltre 12 anni fino a 20 anni	8,00 %
		oltre 20 anni	12,50 %

Internal Models

- At a global level the *VaR* is calculated, $VaR = V \times F^{-1}(\alpha)$ is the quantile of the (€) return distribution of the portfolio
- The VaR is a 10 days 99%.
- The Bank can apply for the validation of generic vs. specific risk cross the main asset classes: interest rate, equity, forex, ...
- The capital requirement is not simply 10d-99% VaR, but

$$Capital\ Requirement = MAX(VaR_t,\ \beta \times VaR_M)$$

Where

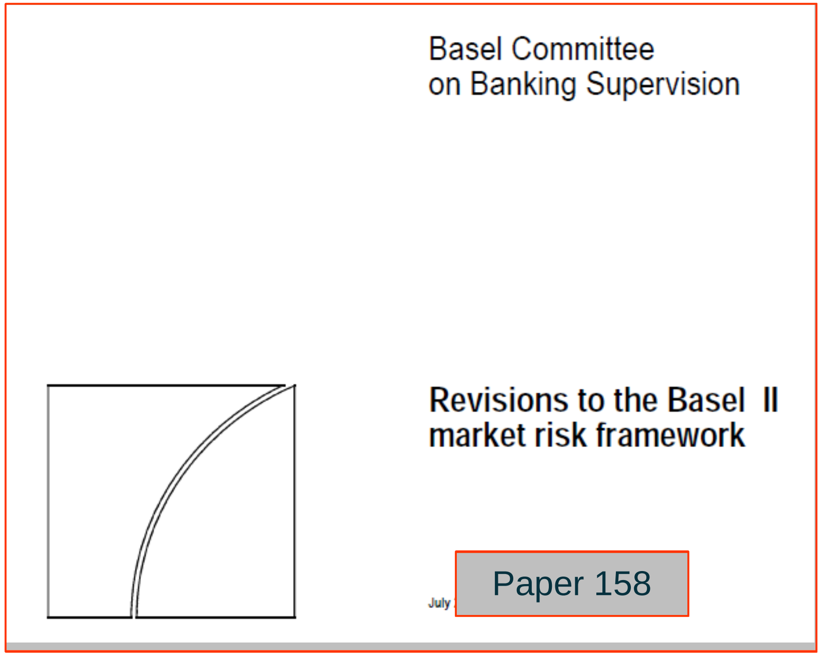
VaR_t and VaR_M are respectively the last and the average VaR of the period (quarter)

$\beta = (3 + x)$, where x depends form the *backtesting* properties of the VaR, e.g. how many times the P&L exceeds the VaR over 1 year of daily data. Look at the below table.

Numero di scostamenti	**Fattore di maggiorazione**
meno di 5	0,00
5	0,40
6	0,50
7	0,65
8	0,75
9	0,85
10 o più	1,00

From Basel 2 to Basel 2.5

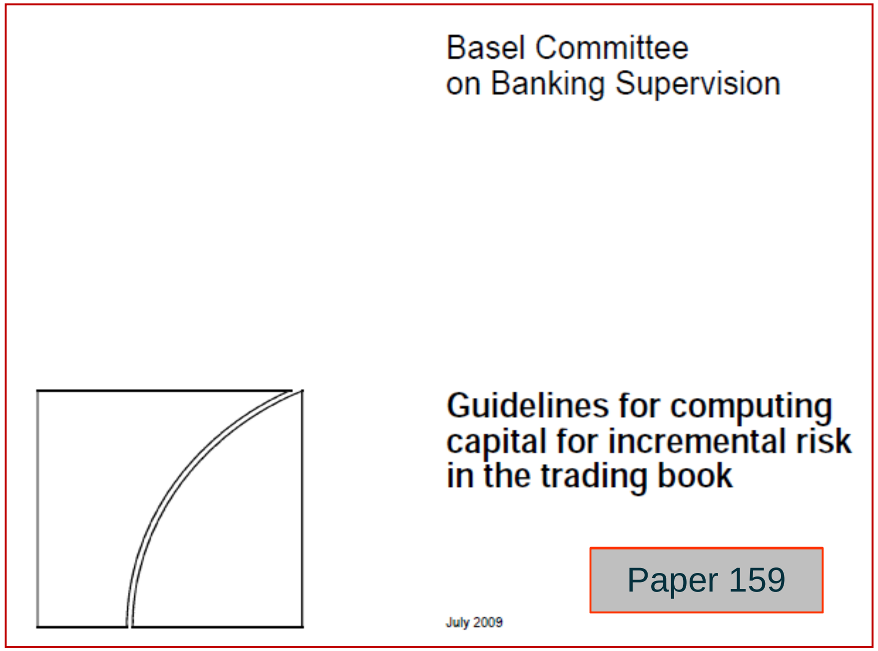
After the first phase of the crisis (2008-2009) a first response was the so called Basel 2.5 reform, that is a revision of the market risk capital requirements. Below the two seminal Basel papers, that came in force by the CRDIII (Capital requirement Directive III) of the European Union on january, 2011.


Basel Committee
on Banking Supervision

Revisions to the Basel II market risk framework

July

Paper 158


Basel Committee
on Banking Supervision

Guidelines for computing capital for incremental risk in the trading book

July 2009

Paper 159

The general consensus after the crisis was that the B2 framework did not capture some **sources** of risk of the trading book (e.g. default risk of bonds) or **extreme** events. Then 2 new risks measures were stated:

- *StressedVaR*, a VaR calculated over a (at least 3 years) period of stress in the markets, w.r.t. the Bank actual porfolio.
- *IRC*, **I**ncremental **R**isk **C**harge, the risk of losses (mainly in bond portfolios) due to default and migration event. It must me calculated with 1 year horizon 99.9% confidence level, to make it comparable with the credit risk set up. Infact we recall that the credit risk capital measure is a stylized VaR (infinite granularity, 1 background risk factor) that aims to mimic a «structural» approach. It was defined by Gordy in the first 2000's.

The 2 new risk measures pose several hard challenges. Example: how to check (and to monitor) the time window where we calculate the StresssedVaR? To select a time frame with some «black Friday» would be a quite stupid approach. We must work w.r.t. to the **bank exposures**. The bank could be *delta short*, *vega long*..differently over its sub portfolios....

Basel 2.5 DrawBacks

The main weakness of the Basel 2.5 is the new capital requirement formula for the banks with the internal models (from Circ.263 Bank of Italy)

$$C_t = \max[VaR_{t-1}; \beta_c \overline{VaR}] + \max[sVaR_\tau; \beta_s \overline{sVaR}]$$
$$+ \max[IRC_\tau; \overline{IRC}] + \max[APR_\tau; \overline{APR}; APR\ Floor]$$

Briefly, ***Double Counting***!! Infact

- VaR «+» SVaR means to measure twice the same risk, the first one with current parameters, the second one with a stressed version
- IRC wants to capture also the migration (downgrade) risk, but it is partially already embedded in the specific issuer risk, with the 10days spread movements.

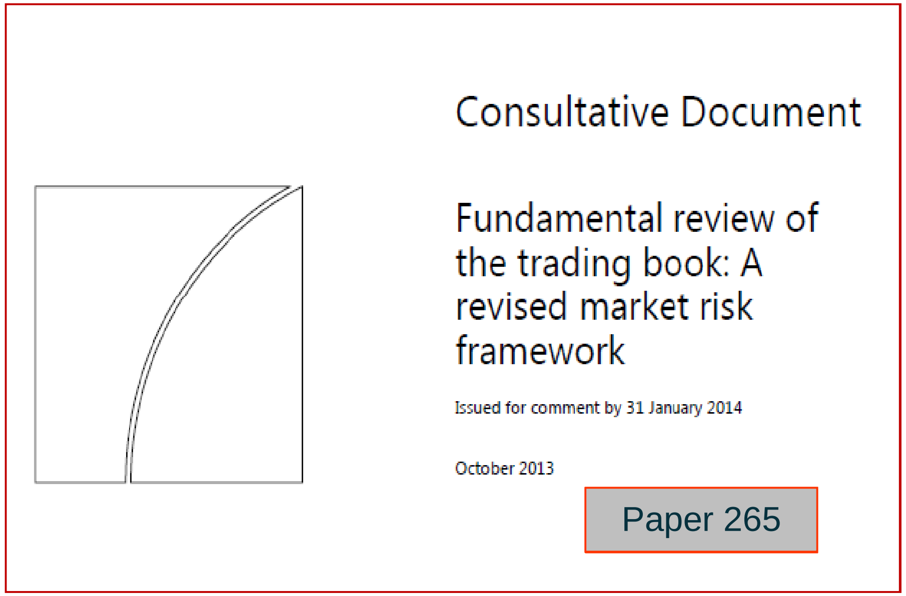
Moreover, the 2.5 reform did not penalize the standard model, except an increase in the equity specific risk (from 4% to 8%). Hence we observed a paradox. The banks that have invested a lot of time and money in quantitative (internal) models had a capital charge **gretaer** than the banks that adopted the very raw standard models

CREDITOTREVIGIANO

B the FRB Review

The FRTB reform

Because of the general criticism about Basel 2.5, new studies started. In 2014 december, the BCBS issued the third version of the *fundamental review of the trading book*. Some QIS (Quantitative impact studies) were performed in last years to test and to calibrate the new reform. The new consultative steps has its deadline on february, 20. Then we will have the official version. The BCBS wrote it wants «*to publish the final revised Accord text within an appropriate time frame*». It could come into force (EU regulation) on 2017-2018. Below the 2 main papers

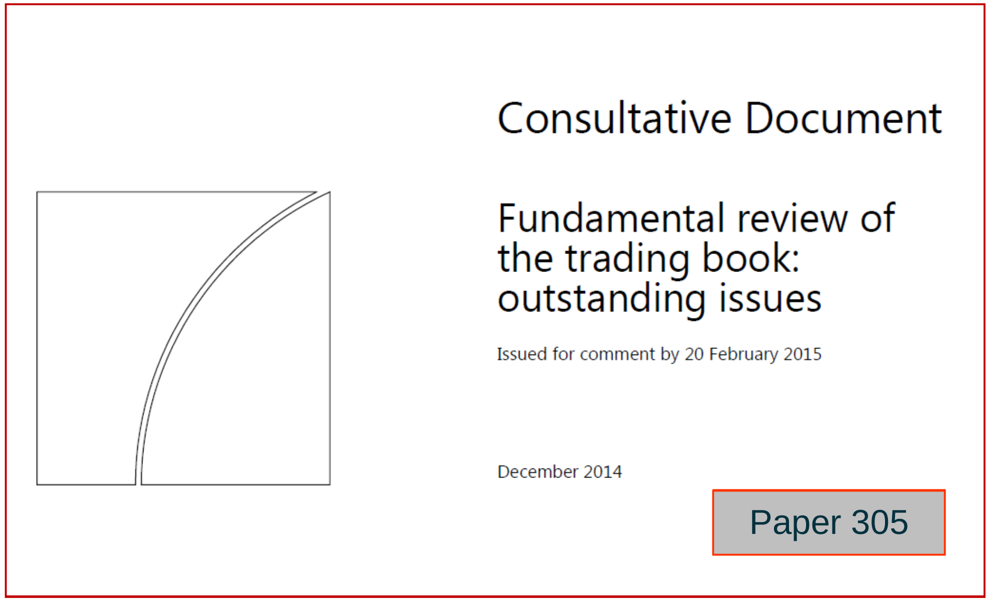

Consultative Document

Fundamental review of the trading book: A revised market risk framework

Issued for comment by 31 January 2014

October 2013

Paper 265


Consultative Document

Fundamental review of the trading book: outstanding issues

Issued for comment by 20 February 2015

December 2014

Paper 305

For a detailed review of the FRTB, see (Bonollo, www.finriskalert.it). Let us summarize the main innovation points:

Metrics

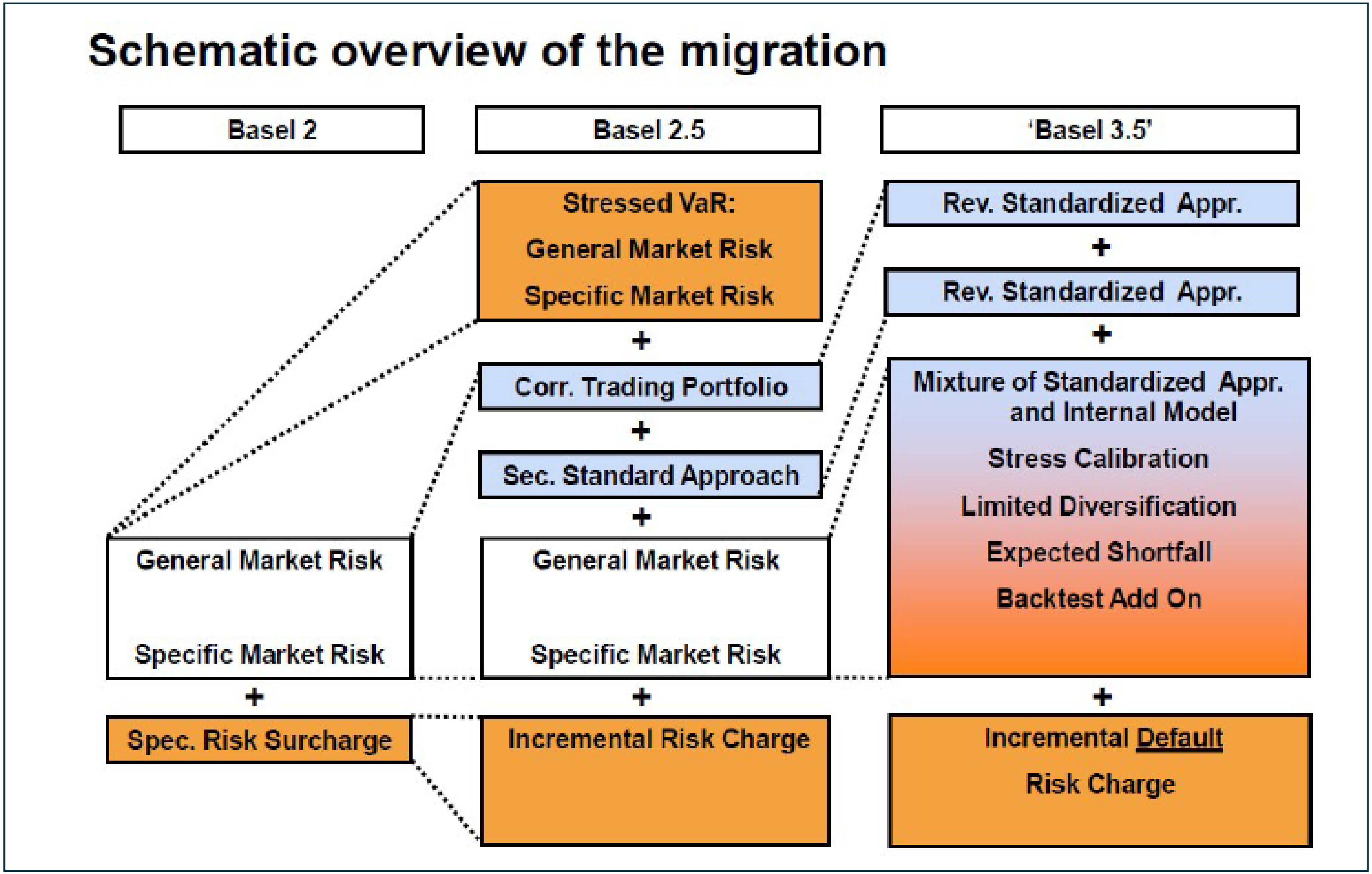
- *Stressed VaR* was **canceled**, raplaced by the general principle of taking in to account an adequate time frame for stressed periods
- The IRC has been **replaced** the the *IDR*, *Incremental Default Risk*, with **only** the default effect
- VaR is replaced by a **97.5**% Expected shortfall (ES)
- The *10days* horizon is now flexible
- *Standard Models*. More sophisticated, with a more granular segmentation of risk weights and several correlation matrix for the *diversification* effects.

→ The **double counting effect disappeared**

Process

- A more complete definition of the boundary between *trading book* (= market risk) vs. *banking book* (= credit risk). More constraints on the switch to **avoid arbitrage**
- More granular validation process (*desk level*)
- In the backtesting procedures (accuracy out of sample, forecasting properties of the risk measures) focus on the *P&L attribution*
- Effectiveness of the reporting process in the desk trading lifecycle

The FRTB – a schema (*)



(*) Quell P. (2014), "FRTB: transition from Basel 2.5 to Basel 3.5", *FRTB Marcus Evans workshop*.

FRTB – Standard Models. Equity Example

- The calculation is based on **sensitivities** (exposures, by the bank), **risk weigts (**by the BCBS) and **correlation matrices** (by BCBS). Below the general formula

$$K_b = \sqrt{\sum_i RW_i^2 MV_i^2 + \sum_i \sum_{j \neq i} \rho_{ij} RW_i MV_i RW_j MV_j}$$

Bucket number	Risk weight (percentage of equity price)
1	55
2	60
3	45
4	55
5	30
6	35
7	40
8	50
9	70
10	50
Residual bucket	70

60. Sensitivities should first be assigned to a bucket according to the buckets defined in the following table:

Bucket number	Size	Region	Sector
1	Large	Emerging market economies	Consumer goods and services, transportation and storage, administrative and support service activities, utilities
2			Telecommunications, industrials
3			Basic materials, energy, agriculture, manufacturing, mining and quarrying
4			Financials including gov't-backed financials, real estate activities, technology
5		Advanced economies	Consumer goods and services, transportation and storage, administrative and support service activities, utilities
6			Telecommunications, industrials
7			Basic materials, energy, agriculture, manufacturing, mining and quarrying
8			Financials including gov't-backed financials, real estate activities, technology
9	Small	Emerging market economies	All sectors
10		Advanced economies	All sectors

71. The correlation parameters γ_{bc} applying to sensitivity or risk exposure pairs across different non-residual buckets are set out in the following table:

Buckets	1	2	3	4	5	6	7	8	9	10
1	–	15%	15%	15%	10%	10%	10%	10%	10%	10%
2	15%	–	15%	15%	10%	10%	10%	10%	10%	10%
3	15%	15%	–	15%	10%	10%	10%	10%	10%	10%
4	15%	15%	15%	–	10%	10%	10%	10%	10%	10%
5	10%	10%	10%	10%	–	20%	20%	20%	10%	15%
6	10%	10%	10%	10%	20%	–	20%	20%	10%	15%
7	10%	10%	10%	10%	20%	20%	–	20%	10%	15%
8	10%	10%	10%	10%	20%	20%	20%	–	10%	15%
9	10%	10%	10%	10%	10%	10%	10%	10%	–	10%
10	10%	10%	10%	10%	15%	15%	15%	15%	10%	–

A Expected Shortfall Backtesting

- Under simple assumptions, to backtest the VaR is quite simple.
- If se assume that the returns are (at least) independent then for each day the probability thet the P&L excess (break) the quantile level is exactly α.
- Then, we can run a classical statistical test for a *binomial* random variable, where
  - We count the *excesses* (usually over a 250 days period)
  - Our *Null Hypothesis* is H_0: Prob(P&L < VaR) = α
  - By the binomial table or normal approximation we get the rejection table. BCBS defined a penality as below
  - Many other extensions in the literature

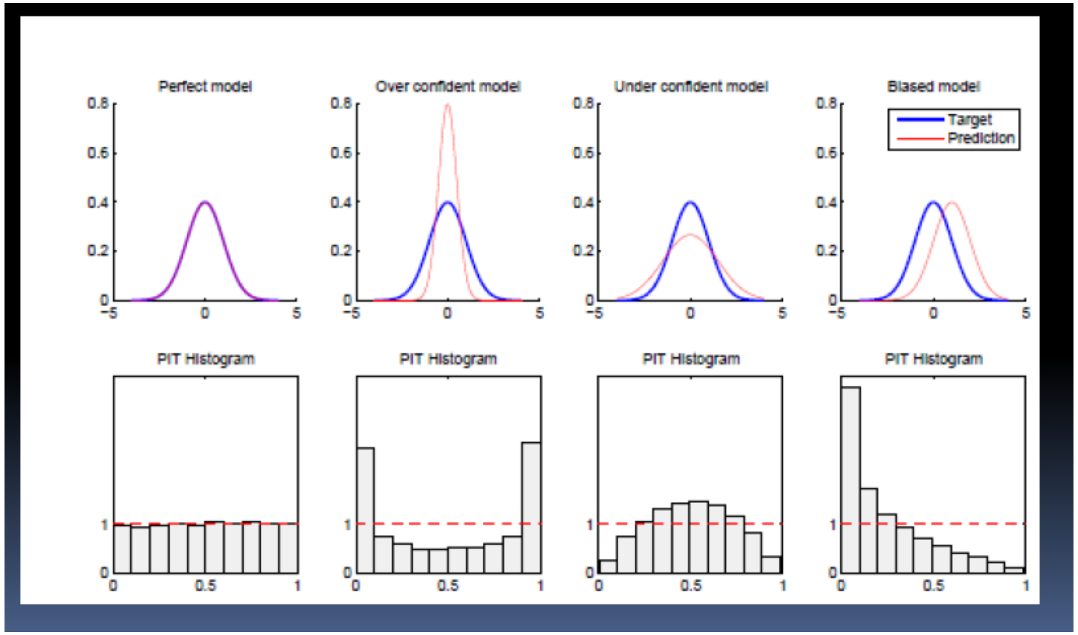
# BREACHES	
GREEN	0-4
AMBER	5-9
RED	10+

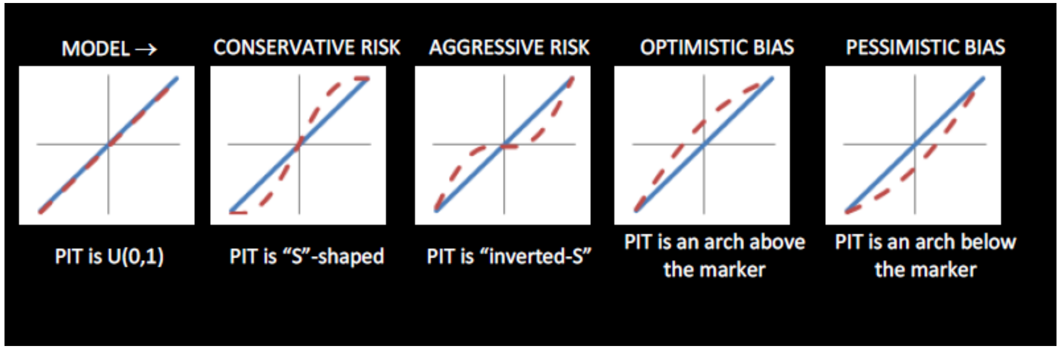
Can we backtest ES?

- From an general perspective, the ES backtesting is more «abstract»
- Day by day, we compare P&L with **what**? In other terms, if we had each day the same ES we «could» compare the empirical returns distribution with the ES level, but in the day by day process I can not test $P\&L_t$ vs. ES_t.
- That is why also in the BCBS remaks it was told «*ES... Is not a elicitable measure*». «To elicit» means:
  - To evoke
  - To extract
  - To give rise to ..
- Which strategy? THE BCBS paper 265 suggests:
  - **Enforcement** of the P&L attribution check, in order to select the eligible desks
  - A **combined backtesting** on both 97.5% and 99% VaR
  - Hence the backtesting would be based on different metrics w.r.t to the reporting risk measure

Yes, we can .. 1

- We can observe «where» the P&L occurs, see the picture
- We compute the *PiT* Probaility (*) integral Transform going back to a U[0,1] situation, by comparing the histogram with the theoretical distribution
- Given the independence and by accumulating the results (e.g.250) we can build the statistical test. Is or not the sample drawn for a U[0,1] random variable? We can run several test, from KS to χ^2…





(*) Quell P. (2014), "FRTB: transition from Basel 2.5 to Basel 3.5", *FRTB Marcus Evans workshop*.

Yes, we can .. 2

- In their very recent paper, Acerbi et al (2014) re-state in a rigoruous framework the problem of the *elicitability* and show how to test in a reliable way the backtest.
- We recall that elicitability simply means that a statistics minimizes a *score* function. *Mean*, *median*, *quantiles* are elicitable, ES is not, this generated a debate about «can we backtest the ES?»
- Some tricks (*). Strategy 1 = VaR & ES jointly.

$$\mathbb{E}\left[\frac{X_t}{ES_{\alpha,t}}+1\middle|X_t+VaR_{\alpha,t}<0\right]=0$$

If $VaR_{\alpha,t}$ has been tested already we can separately test the magnitude of the realized exceptions against the model predictions. Defining $I_t=(X_t+VaR_{\alpha,t}<0)$, the indicator function of an α–exception, we define the test statistics.

$$Z_1(\vec{X})=\frac{\sum_{t=1}^{T}\frac{X_tI_t}{ES_{\alpha,t}}}{N_T}+1 \tag{4}$$

For this test we choose a null hypothesis

$$H_0: P_t^{[\alpha]}=F_t^{[\alpha]},\ \ \forall t$$

where $P_t^{[\alpha]}(x)=\min(1,P_t(x)/\alpha)$ is the distribution tail for $x<-VaR_{\alpha,t}$. The alternatives are

$$\begin{aligned}H_1:\quad & ES^F_{\alpha,t}\geq ES_{\alpha,t},\ \text{ for all } t \text{ and } > \text{ for some } t\\ & VaR^F_{\alpha,t}=VaR_{\alpha,t}, \text{for all } t\end{aligned}$$

(*) Acerbi C., Szekely B "Backtesting Expected Shortfall", *MSCI Research paper*

Strategy 2 = Backtest ES directly

A second test follows from the unconditional expectation

$$ES_{\alpha,t} = -\mathbb{E}\left[\frac{X_t\, I_t}{\alpha}\right] \tag{5}$$

that suggests to define

$$Z_2(\vec{X}) = \sum_{t=1}^{T} \frac{X_t\, I_t}{T\,\alpha\, ES_{\alpha,t}} + 1 \tag{6}$$

Appropriate hypotheses for this test are

$$\begin{array}{ll} H_0: & P_t^{[\alpha]} = F_t^{[\alpha]}, \ \forall t \\ H_1: & ES^F_{\alpha,t} \geq ES_{\alpha,t}, \ \text{ for all } t \text{ and } > \text{ for some } t \\ & VaR^F_{\alpha,t} \geq VaR_{\alpha,t}, \text{for all } t \end{array}$$

We have again $\mathbb{E}_{H_0}[Z_2] = 0$ and $\mathbb{E}_{H_1}[Z_2] < 0$ (proposition A.3). Remarkably, these results do not require independence of the X_t's. Furthermore, the test can be immediately extended to general, non–continuous distributions, by replacing I_t with

$$I'_t = (X_t + VaR_{\alpha,t} < 0) + \frac{\alpha - Prob[X_t + VaR_{\alpha,t} < 0]}{Prob[X_t + VaR_{\alpha,t} = 0]}(X_t + VaR_{\alpha,t} = 0);$$

see eq. (4.12) in [1].

Test 2 jointly evaluates frequency and magnitude of α–tail events as shown by the relationship

$$Z_2 = 1 - (1 - Z_1)\frac{N_T}{T\,\alpha} \tag{7}$$

Strategy 3 = U[0,1] & Ranks

This proposal (see «yes we can ..1») states a test statistics and a set of Hyptothesis to check if the sample of *forecast* distribution *P( )* applied to the $P\&L_t$ *i.e.* $P_t(P\&L_t)$, is acceptably drawn from a *U[0,1]*. This is true if the model is perfect, i.e. $P_t = F_t$.

We recall that ES_t is an «output» of $P_t(\ )$.

$$\widehat{ES}_\alpha^{(N)}(\vec{Y}) = -\frac{1}{[N\alpha]}\sum_i^{[N\alpha]} Y_{i:N}$$

$$Z_3(\vec{X}) = -\frac{1}{T}\sum_{t=1}^{T}\frac{\widehat{ES}_\alpha^{(T)}(P_t^{-1}(\vec{U}))}{\mathbb{E}_V\left[\widehat{ES}_\alpha^{(T)}(P_t^{-1}(\vec{V}))\right]} + 1$$

$$\begin{aligned} H_0 &: \quad P_t = F_t, \ \forall t \\ H_1 &: \quad P_t \succeq F_t, \ \text{for all } t \text{ and } \succ \text{ for some } t \end{aligned}$$

Good *power* results of the tests are shown by MC experiments for the 3 strategies.

The *power* is the probability to reject properly H1 when it is false.

Acerbi C., Szekely B “Backtesting Expected Shortfall”, *MSCI Research paper*

Conclusions

Evolution or **R**evolution?

- A positive new trade off internal vs. standard models
- ES & internal models. A **deep impact** on reporting, model approval and backtesting procedures
- Standard models. A **revolution** in complexity (hopely in risk sensitiveness). Instrument and risk factors data, mapping, greeks, IT systems. A new owner (Risk Mgt) for the regulatory process